MODELING OF THIN FILM RESONATORS AND FILTERS

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ABSTRACT

Thin film piezoelectric resonators have been used in the synthesis of crystal filters at microwave frequencies and for oscillator control [1-6]. These resonators may be modeled using equivalent circuits or analytical expressions derived from acoustic boundary value excitation problems. Results of the modeling are described in three forms; 1) impedance characteristics plotted versus frequency, 2) analytical expressions of impedance, and 3) lumped element equivalent circuits useful for filter design and analysis. Two filter types are modeled, a simple ladder filter and a more complex stacked crystal filter using acoustically coupled resonators.

I. Introduction

Resonators whose extent is large in all lateral directions compared to the thickness can be modeled analytically as one dimensional electromechanical structures using suitable differential equations. From a solution of the one dimensional plane wave pure mode boundary value problem the electrical impedance of a simple resonator is found to be,

$$Z=\left(\frac{1}{j\omega C}\right)\left(1-K^2\frac{\tan\phi}{\phi}\right) \quad (1)$$

where

$$\phi=\frac{kd}{2}=\frac{\pi}{2}\left(\frac{f}{f_p}\right)$$

The parameters in the equation, as determined by material constants are, $f_p=V/2d$ parallel resonant frequency where V is the wave velocity and d the resonator thickness and K^2 the piezoelectric coupling coefficient given in material constants by,

$$K^2=\frac{k^2}{k^2+1} \qquad k^2=\frac{e^2}{c^E\epsilon^S}$$

The material constants are; e piezoelectric constant, c^E the stiffness constant, and ϵ^S the constant strain dielectric constant. The capacitance is given by $C=\epsilon^S(area)/d$.

When the parameters are determined by measurement of series and parallel resonant frequencies (f_s and f_p) ;

$$K^2=\frac{\phi_s}{\tan\phi_s} \qquad \phi_s=\frac{\pi}{2}\left(\frac{f_s}{f_p}\right)$$

Loss in the resonator equation is modeled by a finite Q through a complex phase,

$$\phi=\frac{\pi}{2}\left(\frac{f}{f_p}\right)(1-j/2Q) \qquad Q_r=\frac{f}{2}\left(\frac{d\phi_z}{df}\right)\bigg|_{f=f_r}$$

Here material Q is essentially equal to the impedance phase slope Q_r measured at resonant frequencies.

Equation (1) is an accurate model of most pure mode resonators that predicts harmonic responses but may not be as convenient to use in design as a lumped element model.

II. Equivalent Circuits

A lumped element circuit model often used by low frequency crystal filter designers is the Butterworth Van Dyke (BVD) model shown in Fig. 1. From an analysis of the circuit an expression for resonator impedance can be obtained in a convenient normalized pole-zero form,

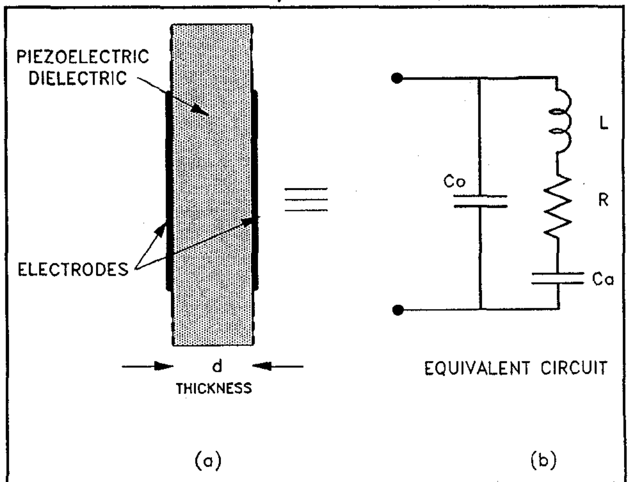

Fig. 1 a) Cross section of piezoelectric resonator and b) Butterworth Van Dyke lumped element equivalent circuit.

$$Z=(1/j\omega C_{lf})z_s/z_p \quad (2)$$

where

$$z_s=1-\left(\frac{f}{f_s}\right)^2+j\left(\frac{f}{f_s}\right)/Q_s$$

$$z_p=1-\left(\frac{f}{f_p}\right)^2+j\left(\frac{f}{f_p}\right)/Q_p$$

$$f_s=\left(\frac{1}{2\pi\sqrt{LC_a}}\right) \qquad f_p=f_s\sqrt{1+\frac{C_a}{C_o}}$$

$$Q_s=\frac{1}{2\pi f_s C_a R} \qquad Q_p=Q_s\sqrt{1+\frac{C_a}{C_o}}$$

From measurements of; f_s, f_p, low frequency capacitance

C_{lf}, resistance at series resonance R, the lumped element quantities are determined as follows,

$$C_o = \left(\frac{f_s}{f_p}\right)^2 C_{lf} \qquad C_a = \left[\left(\frac{f_s}{f_p}\right)^2 - 1\right] C_{lf}$$

$$L = \frac{1}{2\pi f_s C_a} \qquad R = \frac{\pi^2}{8}\frac{X_o}{Q_s K^2} \tag{3}$$

If the impedance equation is used for modeling the resonator, rather than lumped elements in a circuit analysis program, then the Qs in (2) are best determined by phase slope measurements.

The two resonator models can be linked by doing a mathematical expansion of (1) about series resonance and deriving the pertinent quantities,

$$\frac{C_a}{C_o} = 2\left(\frac{2}{\pi}\right)^2 K^2 = 0.81057K^2$$

$$C_{lf} = \frac{C}{1-K^2} = C(1+k^2)$$

or equivalently where C_{lf} is determined by the constant stress dielectric constant ϵ^T and C from the constant strain dielectric constant ϵ^S.

Figure 2 shows a plot of resonator impedance using both (1), solid line, and (2), dashed line. Although the lumped element model does not show the harmonic response it closely approximates the resonator around the resonances used to determine the element values.

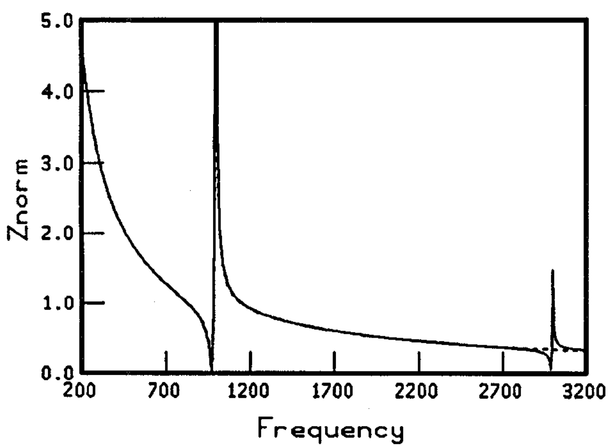

Fig. 2 Normalized absolute value of impedance from equations representing plane wave boundary value solution, solid line, and Butterworth Van Dyke (BVD) equivalent lumped element equivalent circuit model, dashed line. The BVD model does not predict harmonic responses.

III. Composite Resonator

At microwave frequencies crystal resonators are only approximately described by the impedance relations in (1) and (2). Issues such as electrode metal thickness and associated losses are not modeled by simple impedance equations. A useful method is to employ transmission line concepts and matrix algebra to analyze relatively complex geometries. The only uncertainty in this process is properly modeling the piezoelectric plate and its boundary conditions. As illustrated in Fig. 3 the piezoelectric plate is essentially a three port network having two acoustic ports and one electrical port. By solving the boundary value problem an equation for the electrical port impedance in terms of arbitrary mechanical loads is obtained;

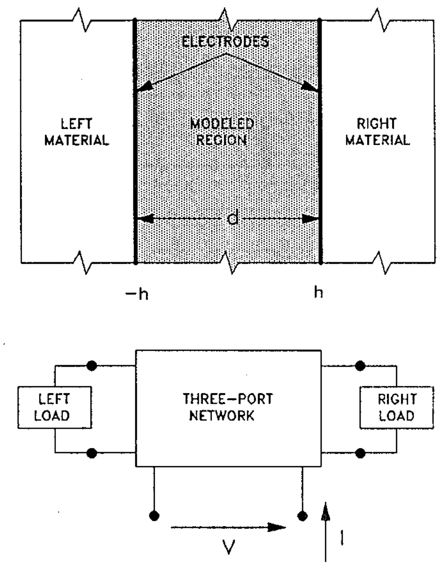

Fig. 3. Schematic of three port configuration of a piezoelectric plate having arbitrary mechanical loads on left and right.

$$Z = \left(\frac{1}{j\omega C}\right)\left(1 - K^2\frac{\tan\phi}{\phi}Z_m\right) \tag{4}$$

$$Z_m = \frac{(z_r + z_l)\cos^2\phi + j\sin 2\phi}{(z_r + z_l)\cos 2\phi + j(z_r z_l + 1)\sin 2\phi}$$

where z_r and z_l denote load impedances normalized by the piezoelectric film impedance, $\phi = \theta/2$, where $\theta = kd$ is the phase across the piezoelectric film.

Composite resonator structures modeled using (4) require that the external region be expressed as an equivalent impedance. For complex geometries the effective load impedance of the substrate can be found from an **ABCD** matrix cascading of equivalent transmission line sections or by successive use of the transmission line equation,

$$Z_{in} = Z_0\left\{\frac{Z_l\cos\theta + jZ_0\sin\theta}{Z_0\cos\theta + jZ_l\sin\theta}\right\} \tag{5}$$

Here Z_{in} is the input impedance, Z_0 the characteristic impedance, $\theta$the phase across the delay section, and Z_l the load impedance attached to the line section.

Using partial fraction expansions on (4) it is possible to derive the Mason circuit model for the one dimensional pure mode plane wave resonator. This circuit, Fig. 4, and variations are used to model piezoelectric transduction problems with varying degrees of success. The model is exact for the one dimensional pure mode case.

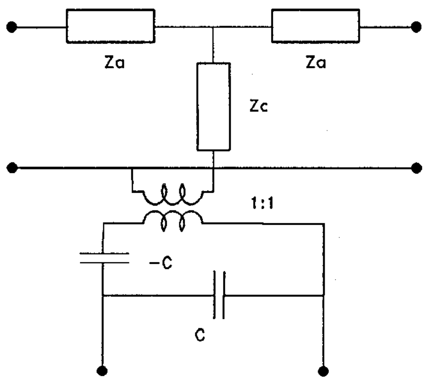

Fig. 4. Mason equivalent circuit model for the piezo-electric plate. $Z_a = jZ_e \tan\phi$, $Z_e = -jZ_e csc\theta$, $Z_e = \pi/(\omega_p C K^2)$.

IV. Filters

Filters using piezoelectric resonators take one of two forms; electrically or mechanically coupled. A simple manifestation of the electrically coupled resonator filter is a ladder filter composed of resonators in the series and/or shunt branches.

Insertion loss of a crystal filter is determined primarily by the series resistance of the crystal as given in (3). The material and resonator quality is the K^2Q product whereas the reactance X_0 is determined by the design procedure. In low frequency quartz crystals the K^2Q product can approach 1000 yet the series resistance is relatively large due to the large capacitative reactance X_0 at low frequencies due in part to the need for small resonator areas. In contrast a thin film aluminum nitride resonator at 1 GHz with a K^2Q of only 300 and a reactance of 50 ohms can yield a low series resistance in a 50 ohm system.

The filter is shown to illustrate the main features of the modeling rather than a particular design. Mid-band insertion loss is determined by resistance of the series resonators at series resonance aided in part by the parallel resonance of the shunt resonator in mid-band. The out of band levels are determined strictly by the capacitive voltage divider ratios and are not determined by electromechanical properties of the resonator. The response of a simple ladder filter, computed using (1) to model the resonators, is shown in Fig. 5.

A great deal of work has been done on ladder filter design and production for low frequencies where the technology is well established.

The stacked crystal filter (SCF), shown in Fig. 6 is a more complex filter using mechanically coupled resonators in each individual section. The device is different than the monolithic filter which employs transverse plate wave propagation to achieve coupling. The tight thickness mode coupling in the SCF gives rise to a rather rounded filter response, shown in Fig. 7 near the center frequency, characteristic of over coupled resonators.

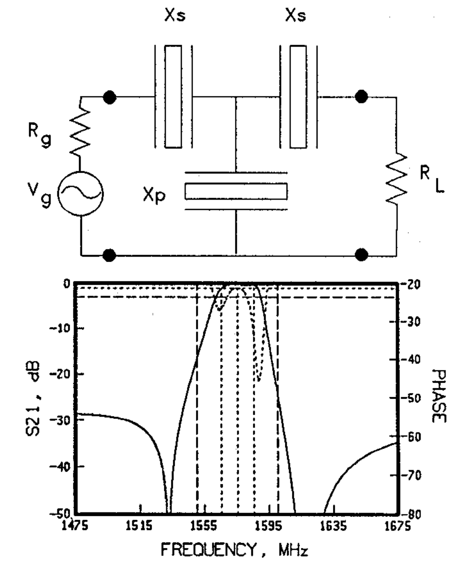

Fig. 5 Simple microwave T ladder filter for GPS using three crystal resonators. The resonator K^2Q is 325.

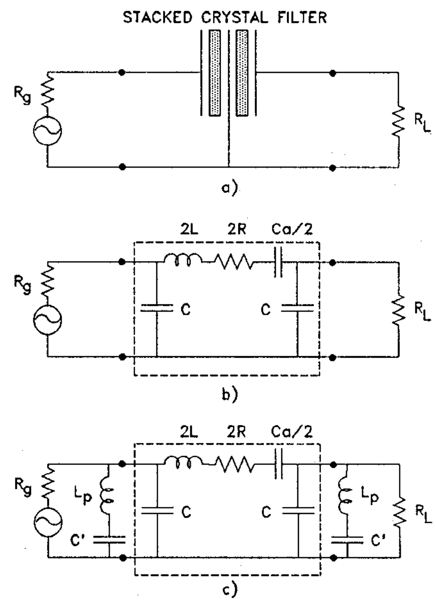

Fig. 6 Stacked crystal filter. a) Schematic representation, b) equivalent circuit derived from Mason model and Butterworth Van Dyke expansions, c) Inductor tuning to increase bandwidth.

An equivalent circuit of the SCF is shown in Fig. 6b for a single section. The circuit model was derived from an expansion of two Mason models joined back to back (just like the construction of the resonator itself) [7]. The final circuit is obtained from an expansion of the distributed impedances to give a lumped element description and the series RLC network. The lumped elements in the series path have twice the impedances of BVD elements in a single resonator. Alternatively this may be viewed as the series branch of a single resonator having half the piezoelectric coupling coefficient. The latter interpretation is consistent with the fact that the SCF is a composite resonator operating with two half wavelengths (i.e. at the second overtone) and therefore has approximately half the coupling coefficient of a fundamental mode resonator.

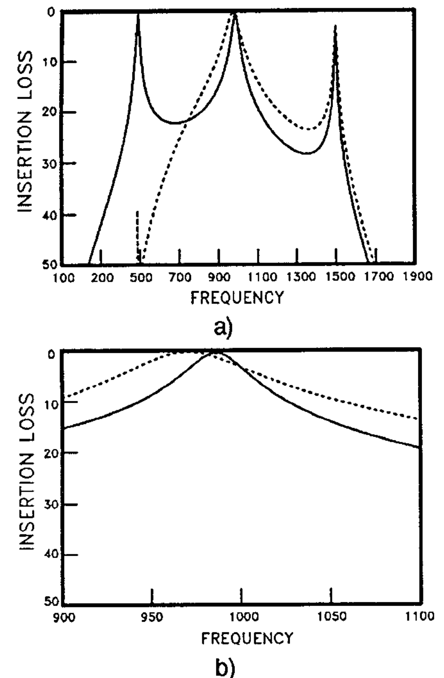

Fig. 7 Computed response of stacked crystal. Solid line without inductor tuning, dashed line with inductor tuning and low frequency trap.

Minimum insertion loss for the single section SCF occurs when the reactance X_o is equal to the source and load resistance. For smaller X_o the shunt current is too large and for larger X_o the series resistance is too high. Under these restrictive conditions the insertion loss is determined by K^2Q alone.

Unlike normal crystal filters the SCF exhibits close in overtone responses. The fundamental frequency, where there is a half acoustic wavelength across the thickness of the resonator, is the lowest frequency response but of narrower bandwidth than the second response that occurs when there is a half acoustic wavelength across each part of the SCF resonator. This is the strongest resonance and hence widest bandwidth filter. The next highest response is the third overtone and is of even narrower bandwidth. Considering the wide band response as the center frequency, then the other (probably undesirable) responses occur at half and three halves center frequency. In contrast a fundamental mode ladder filter at center frequency would have its next response at three times center frequency or twice as far away as in the SCF.

The problems of limited bandwidth, insertion loss, and spurs in the SCF can be cured to a certain extent by employing inductor tuning in a similar manner employed in low frequency ladder filters. By using inductors in the shunt branch the shunt current can be reduced relative to the series current without having to use high Q inductors. Once tuning is used the $X_o = R_g$ condition for minimum insertion loss no longer holds and series resistance may be reduced by reducing X_o, by increasing the resonator area. The effect of decreasing X_o and using inductor tuning is illustrated in Fig. 8. Although the bandwidth increase is substantial the realization of inductor Q's of 100 are questionable and the device area is now five times larger.

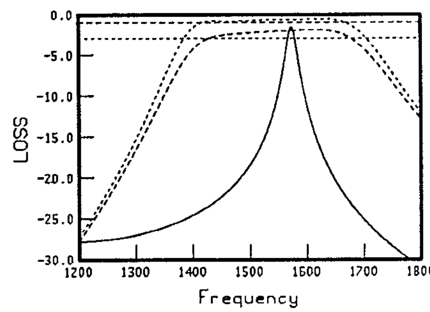

Fig. 8 Mid-frequency range response for a two section stacked crystal. The horizontal dashed lines are the -1 dB and -3 dB references. The solid curve is the SCF with 10 ohm capacitative reactance resonators and 50 ohm source and load resistances without inductor tuning. The two dashed line responses are for inductor Q's of 100 and 25.

V. Summary

Thin film resonators and filters have been modeling using a combination of impedance equations derived from wave propagation boundary value solutions or associated lumped element circuit models and matrix manipulation of transmission line sections. Approximate circuit models allow insight for design purposes while more detailed equations and procedures allow accurate characterization of designs.

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